

# Delayed Feedback Circuits

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# Outline

- What we optimize: SNR
- Delayed Feedback and Conditions Needed to Increase SNR
- Steady-State Analysis
- Experimental Study
- How This Can Fail

# what we want to optimize

$$SNR = \frac{P_s}{\delta P_n}$$

# feedback

- feedback (as shown) preserves SNR
- sharpens linewidth (increases loaded Q) with amount adjustable by G
- resonance tunable by shifting phase

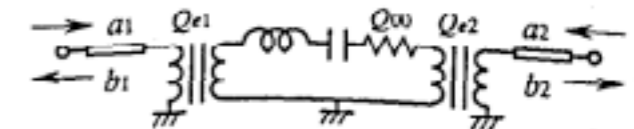
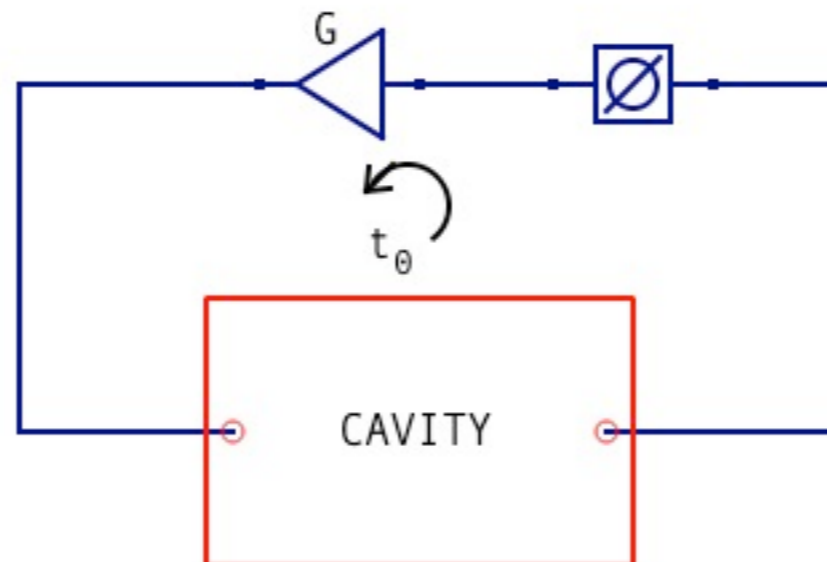


Fig. 2 Two terminal resonance circuit.

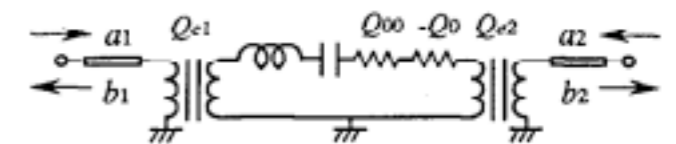
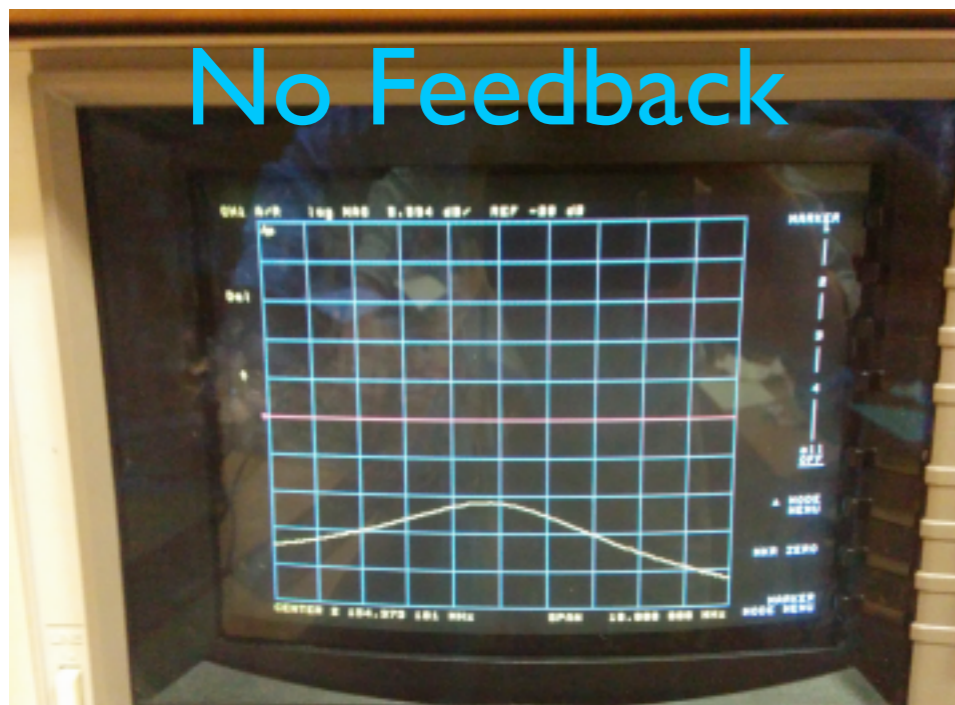


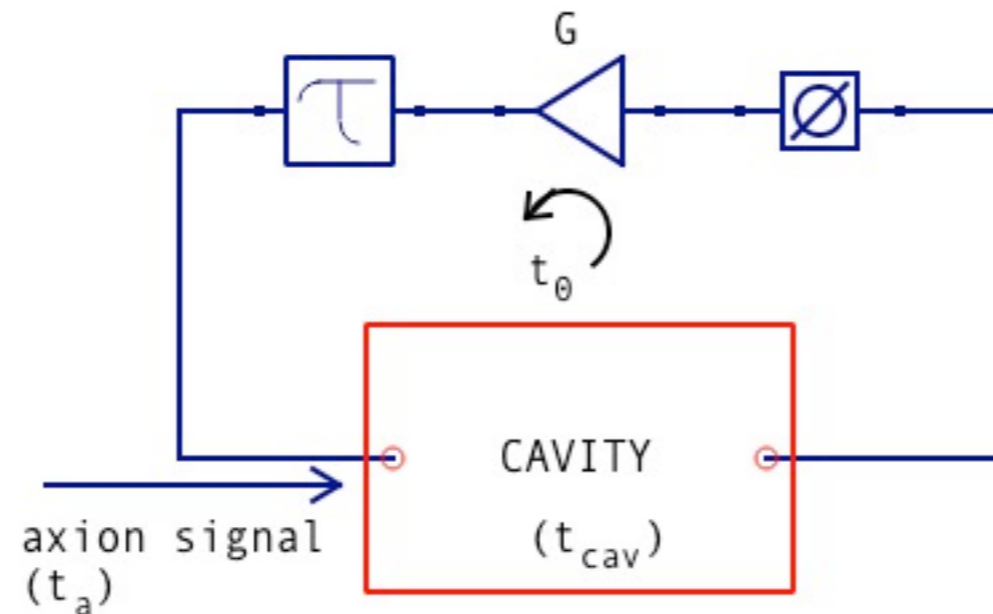
Fig. 3 Two terminal resonance circuit with negative resistance.

$$\alpha = \frac{1}{Q} - GS_{21}$$

$$Q \propto \alpha^{-1}$$



# time-delayed feedback



## Conditions for enhanced SNR:

total gain must be less than 1

$$t_a > t_0$$

$$t_0 = NT$$



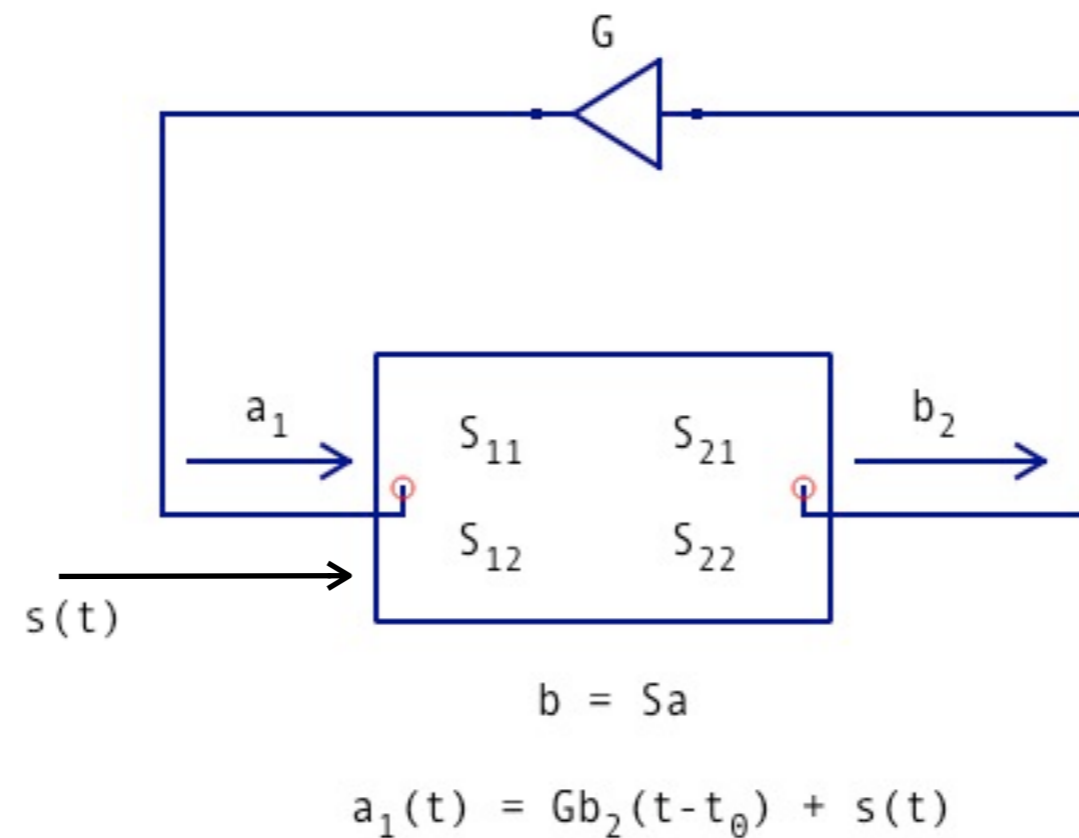
T = period of signal

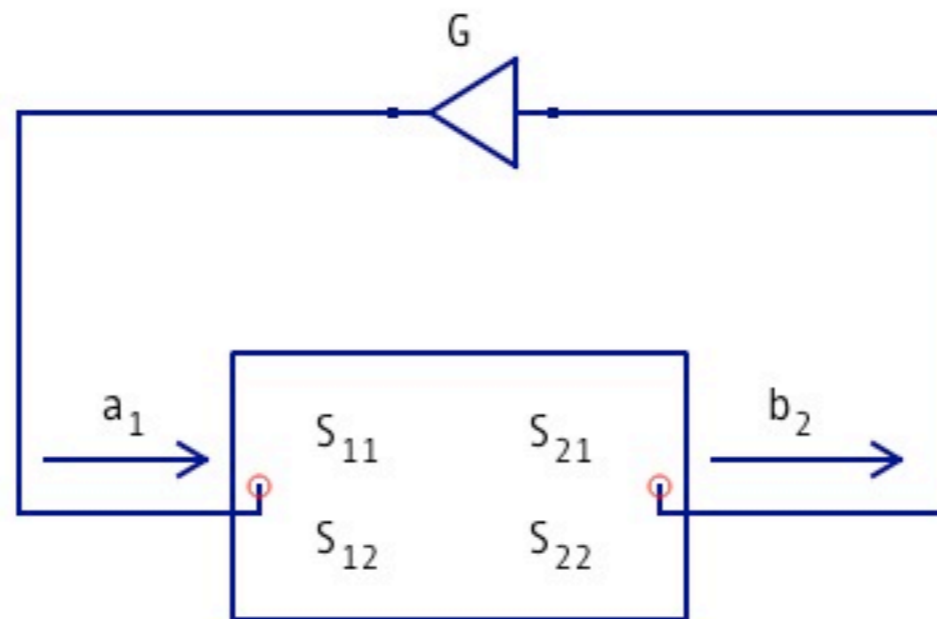
$$t_0 > \frac{1}{2B}$$



from uncertainty principle  
B = bandwidth of thermal noise

# steady-state analysis





$$b = Sa$$

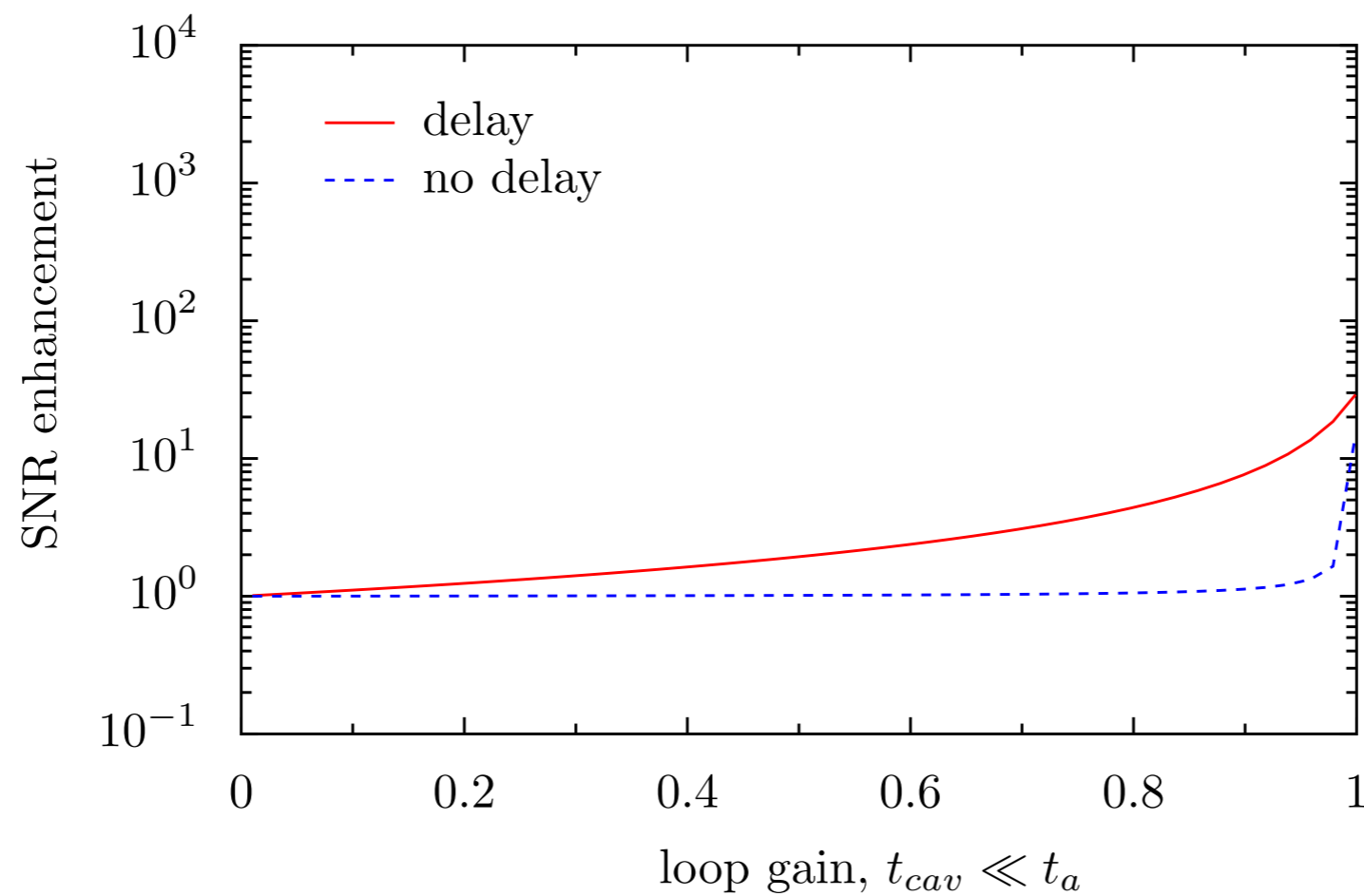
$$Q \propto (1 - GS_{21})^{-1}$$

$$P_{\text{out}} = \langle |b_2|^2 \rangle \propto (1 - GS_{21})^{-2}$$

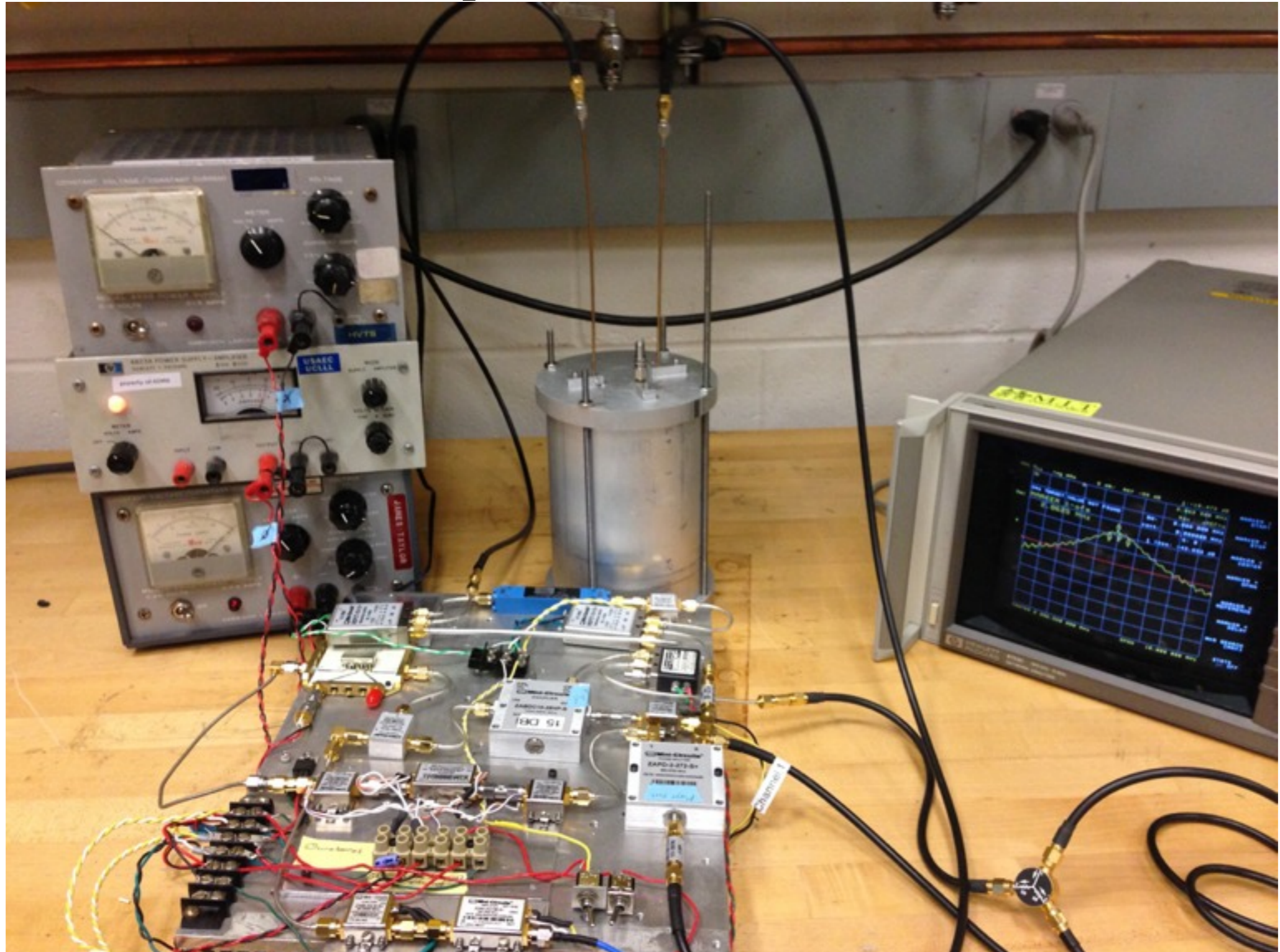
\*assuming  $\langle a_1(t)a_1(t-t_0) \rangle = a_1^2$

$$\begin{aligned} \frac{b_2(t)}{=} & S_{21}a_1(t) \\ & + S_{21}^2Ga_1(t-t_0) \\ & + S_{21}^3G^2a_1(t-2t_0) \\ & + \dots \end{aligned}$$

# predicted enhancement

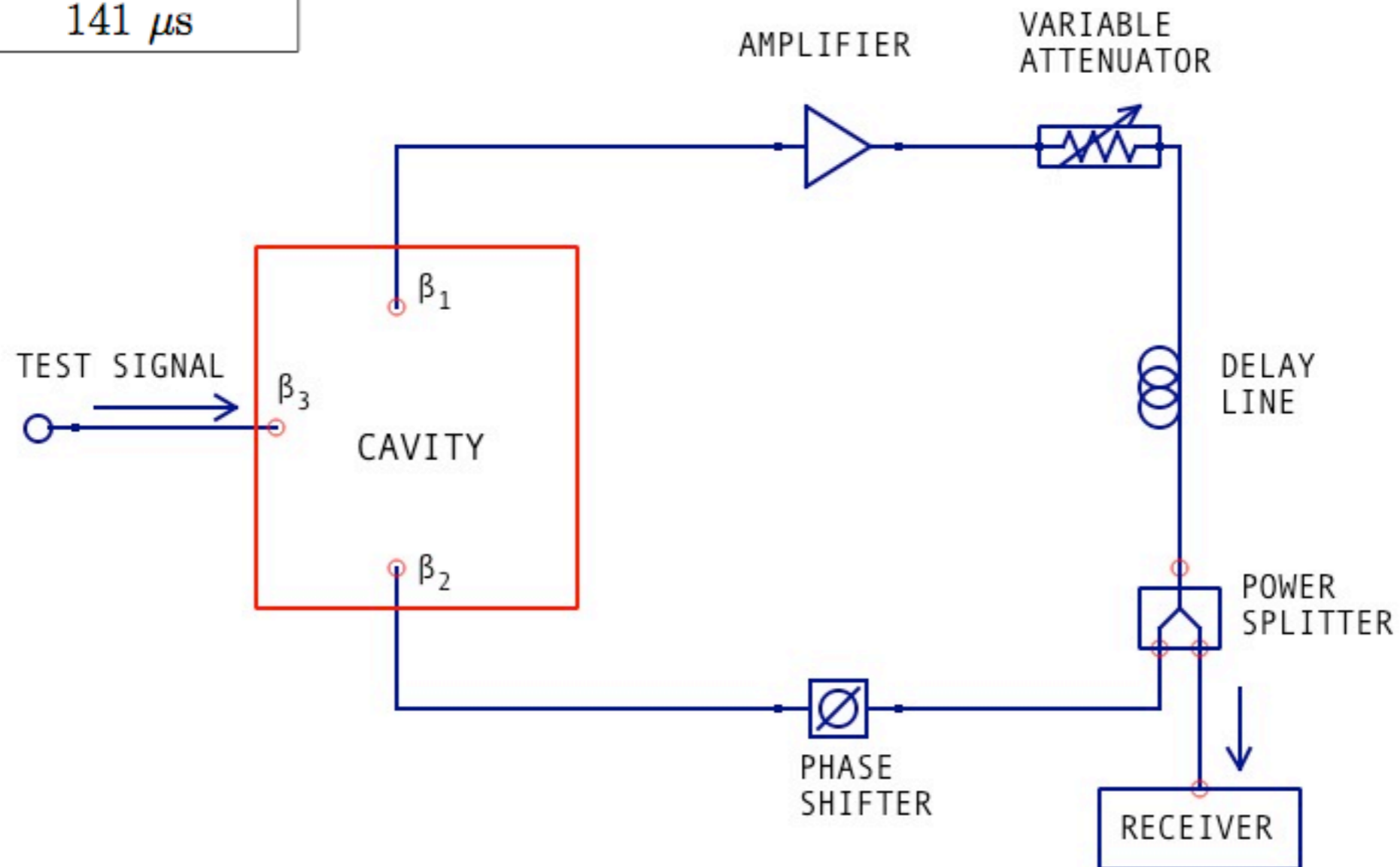


# experiment



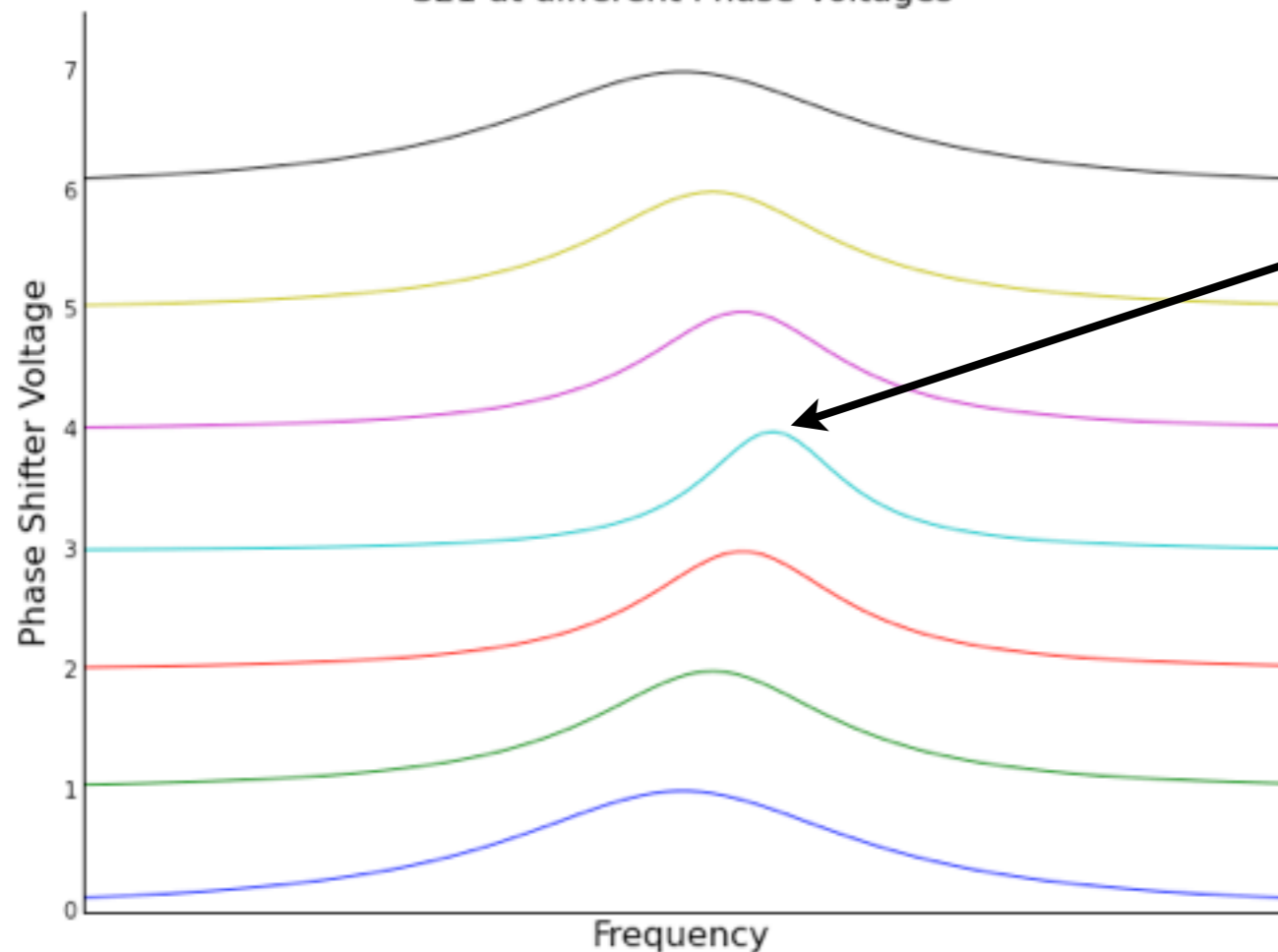
# setup

Parameter	Values
$f_{\text{cav}}$	2.256 GHz
$t_{\text{delay}}$	2.4 $\mu\text{s}$
$t_{\text{cav}}$	170 ns
$t_{\text{axion}}$	141 $\mu\text{s}$



# measurement

S21 at different Phase Voltages



0. Set feedback gain

1. Set phase to maximize Q on N.A.

2. Send in signal at the resonant frequency

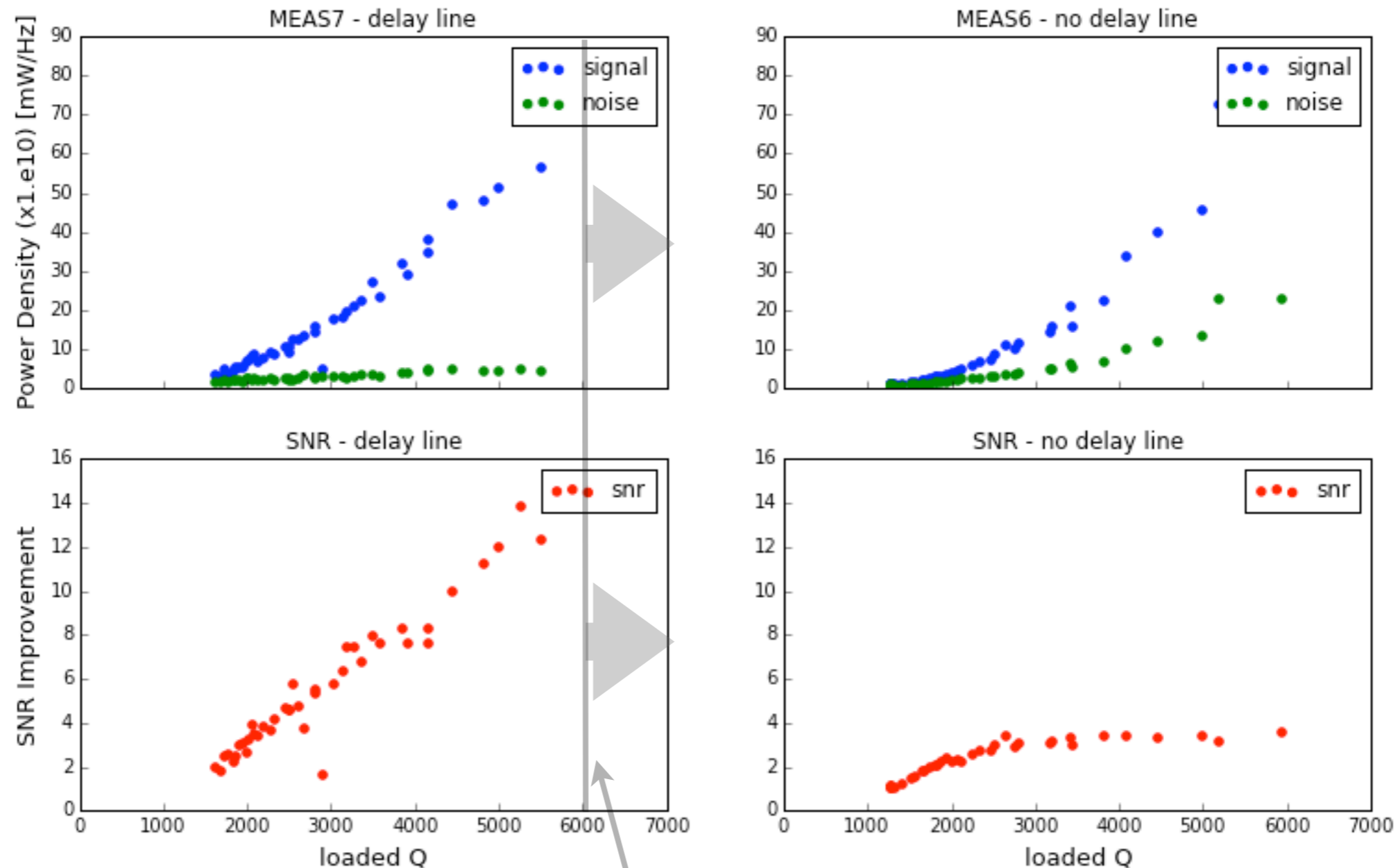
3. Turn N.A. off; measure spectrum analyzer trace at resonant frequency with signal on, signal off to get SNR.

4. Sweep through feedback gain settings.  
Repeat measurement with delay line in.

# network analyzer trace

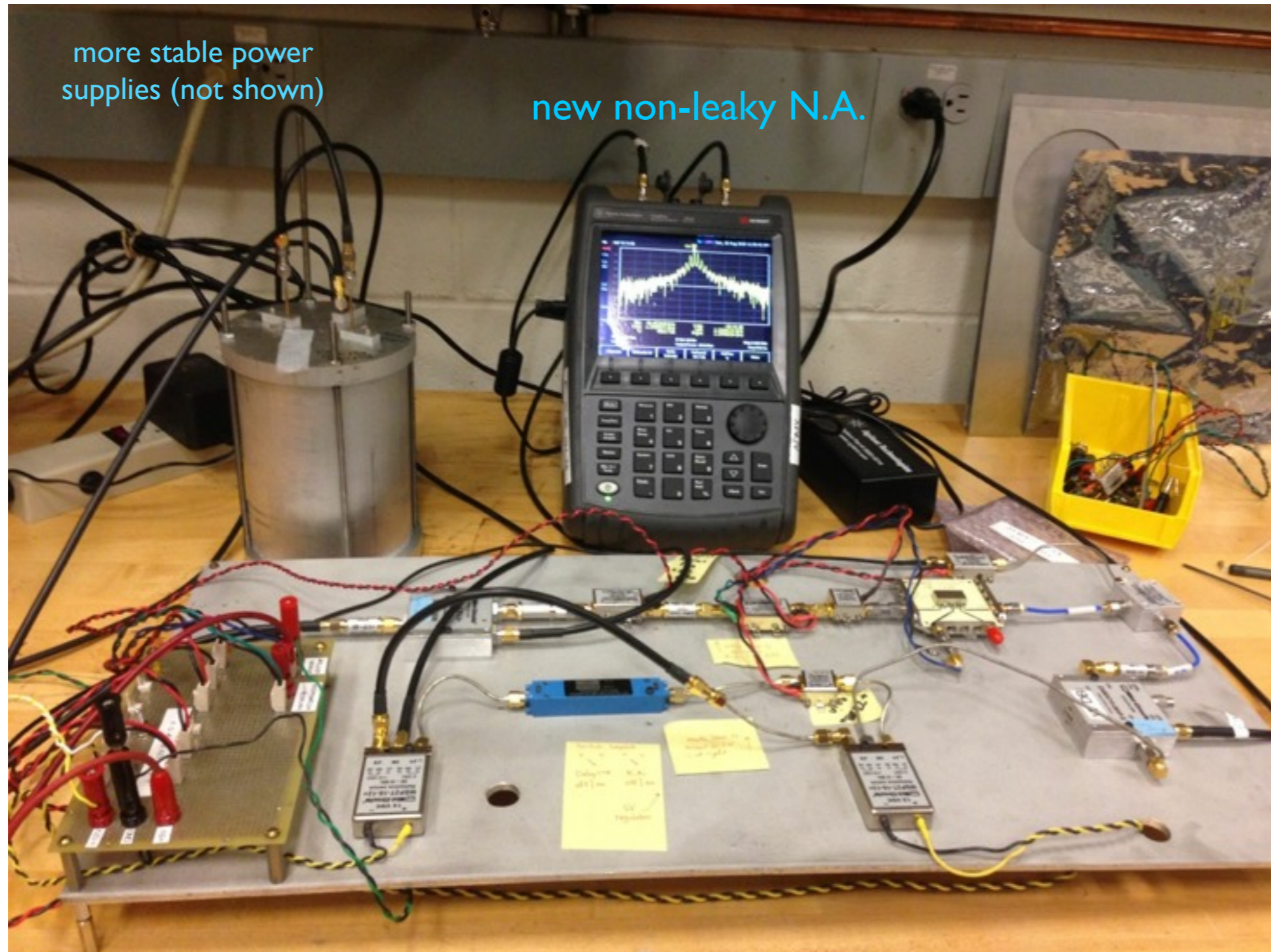


# results



data after this point was cut, as the BW of the fringes was smaller than the resBW of the spectrum analyzer

# status:



# way forward

- calibrating interference fringes
- getting a much smaller delay line/increasing the initial  $Q$  by  $\times 10$  to remove the fringes
- microwave circuit simulations for sanity checking

# problems

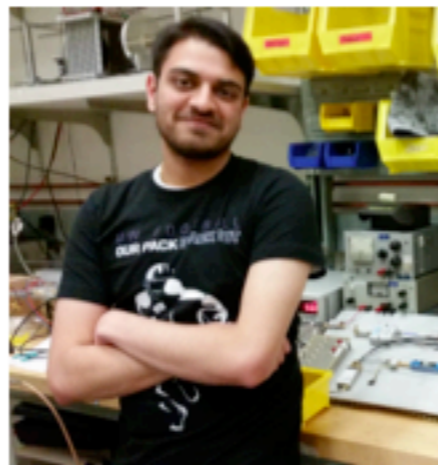
- unintentional external feedback (ie reflections in return line)
- many modes within the feedback bandwidth

# credits

## People

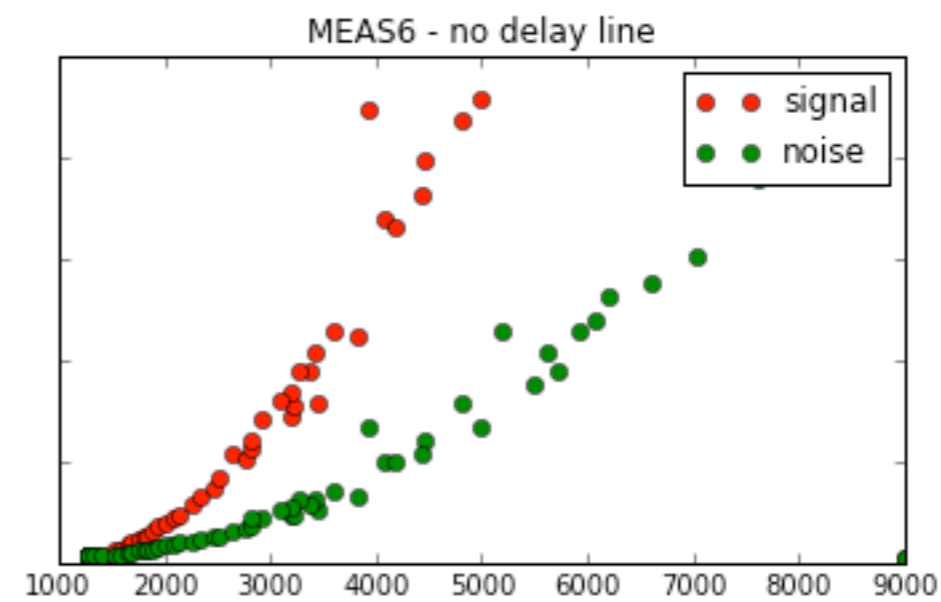
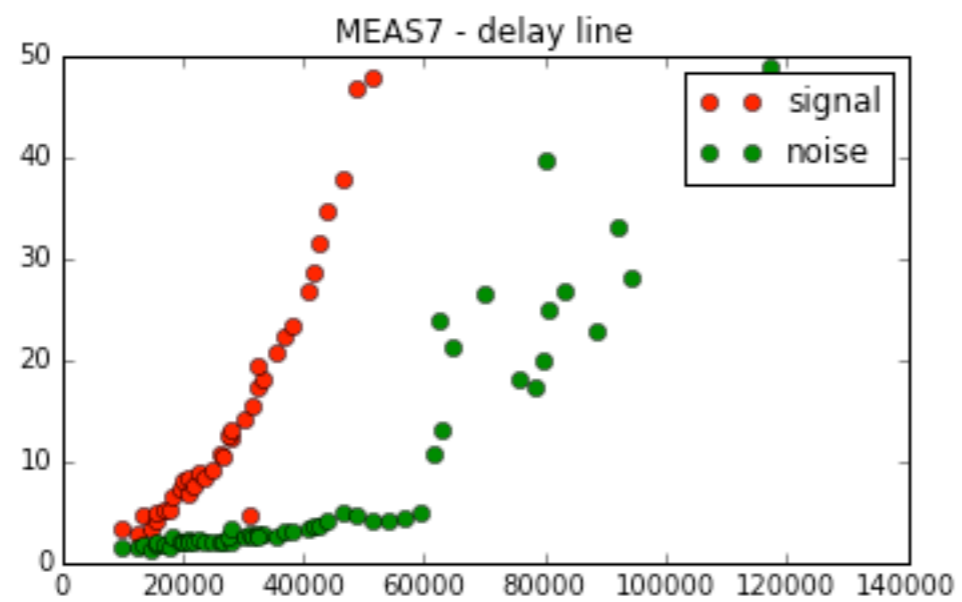
Lisa McBride, Kunal Patel, Gray Rybka

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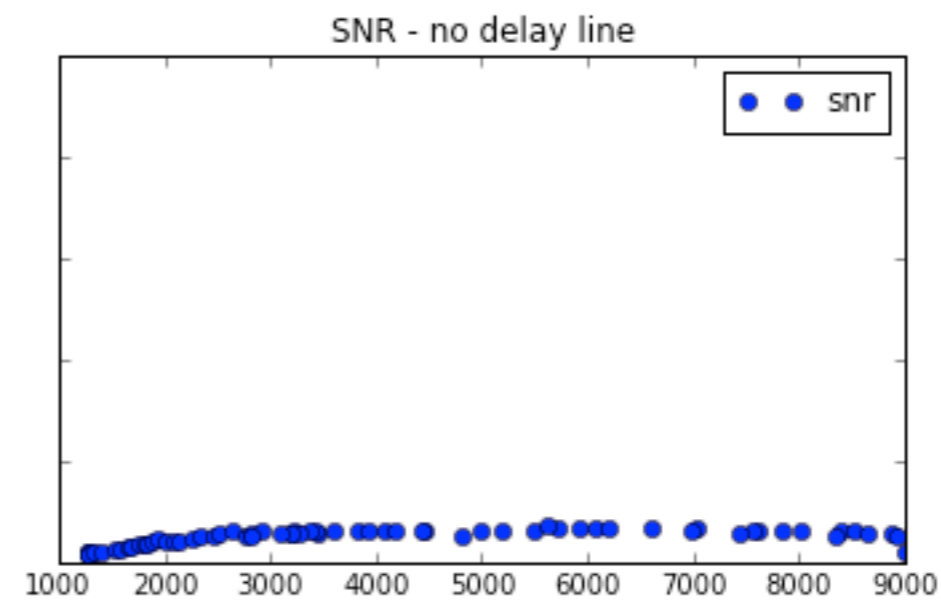
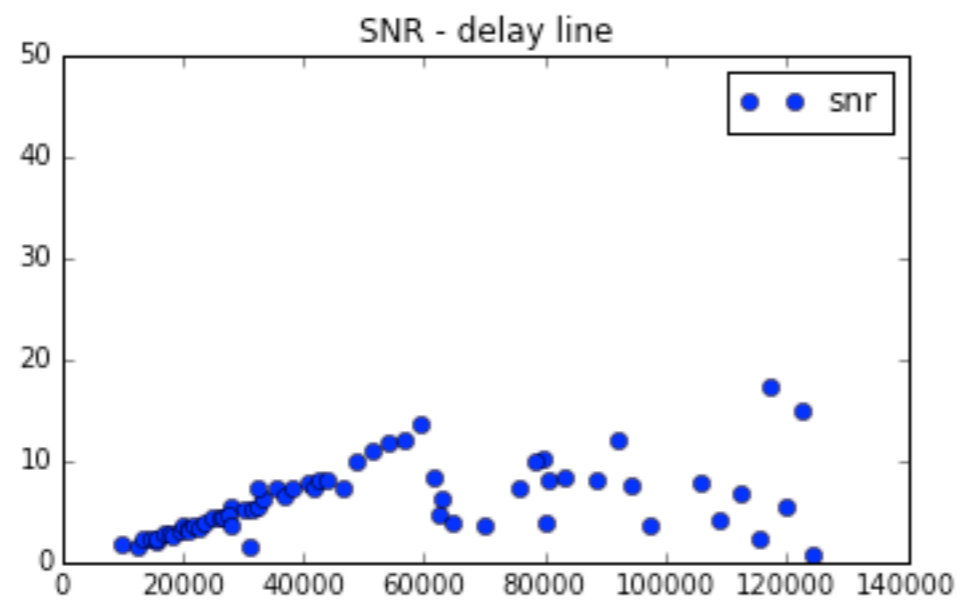


# backup

Power Density

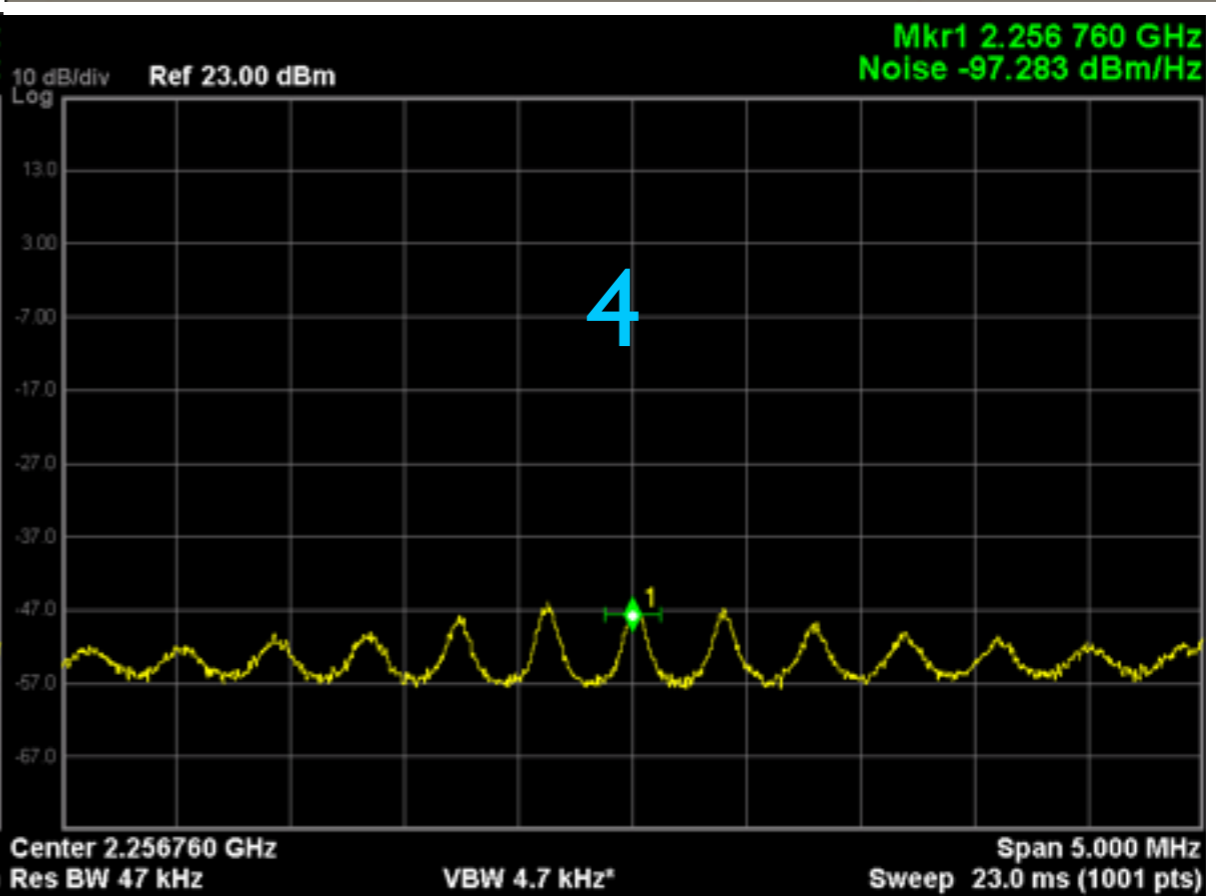
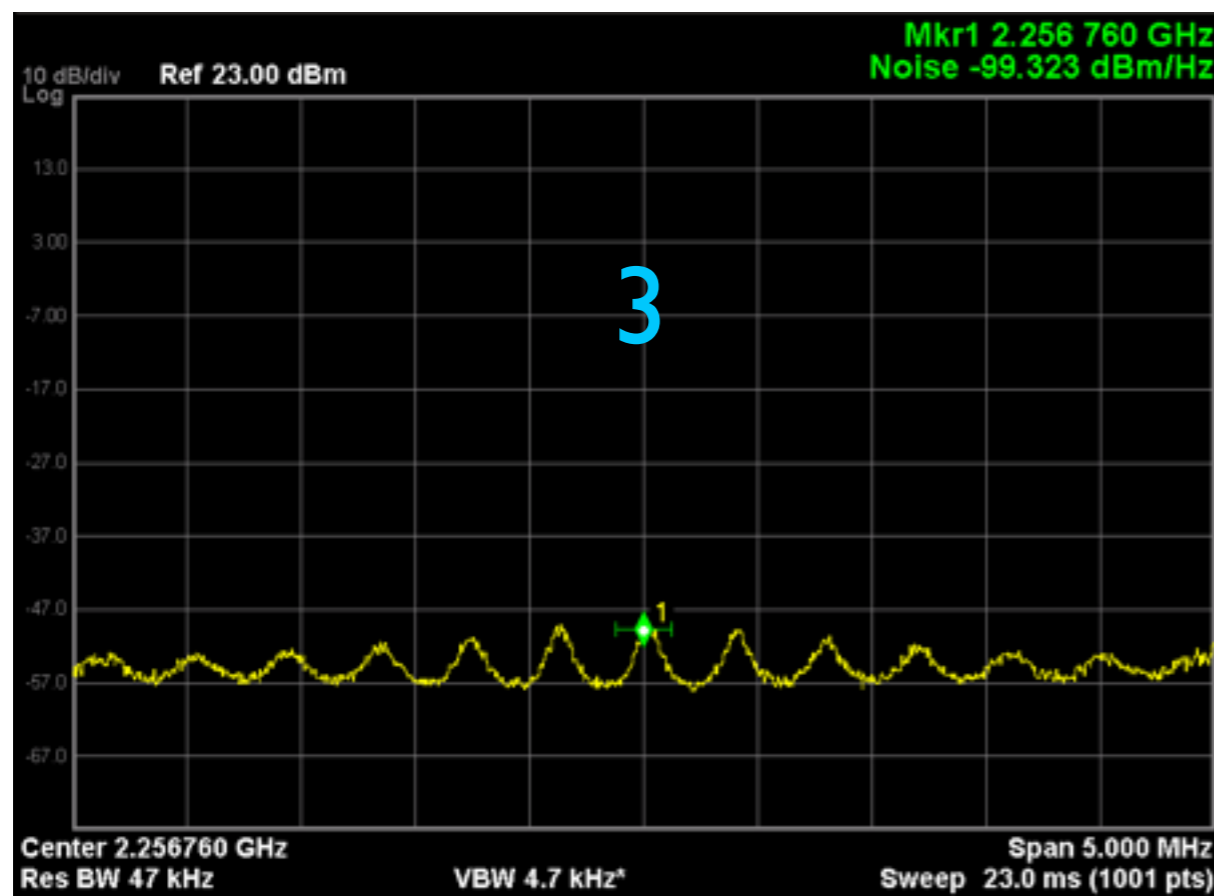
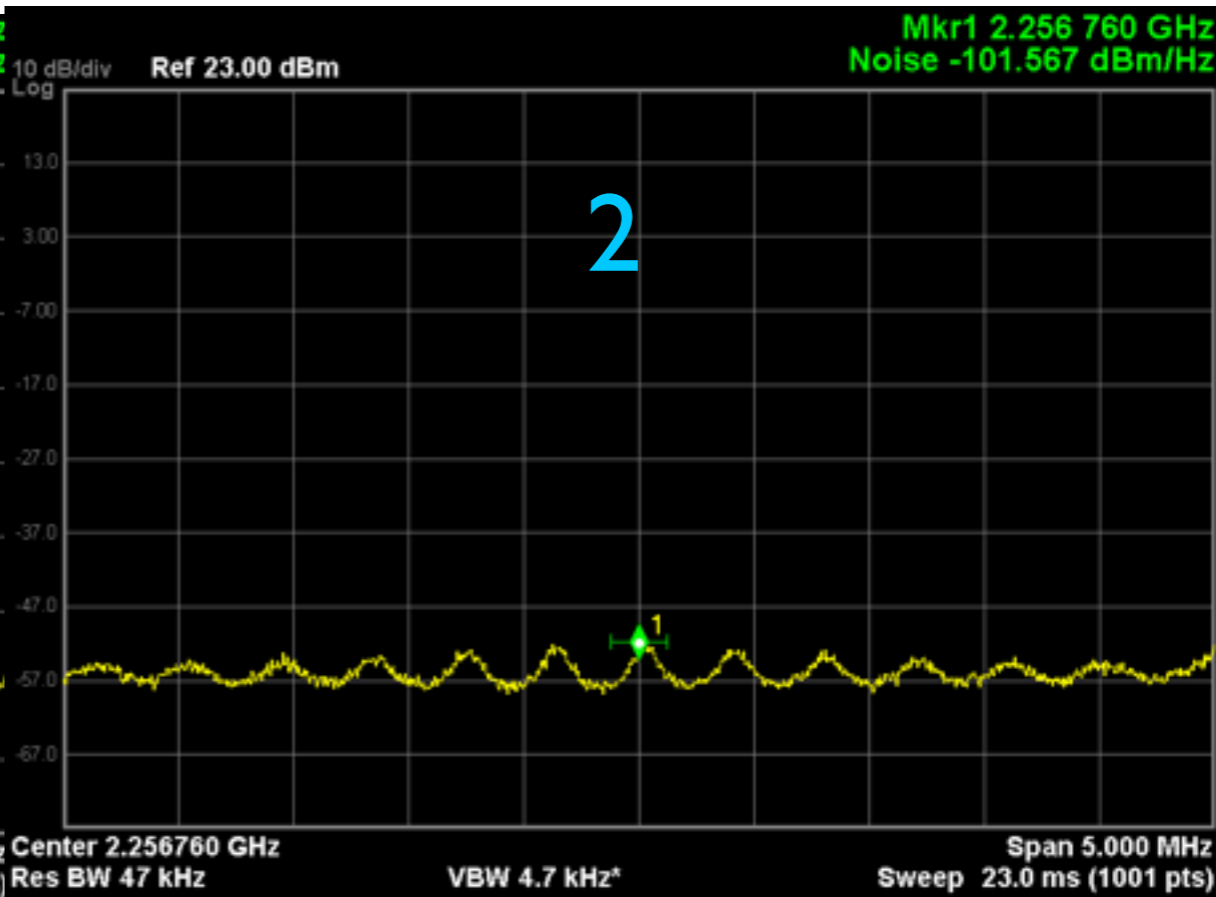
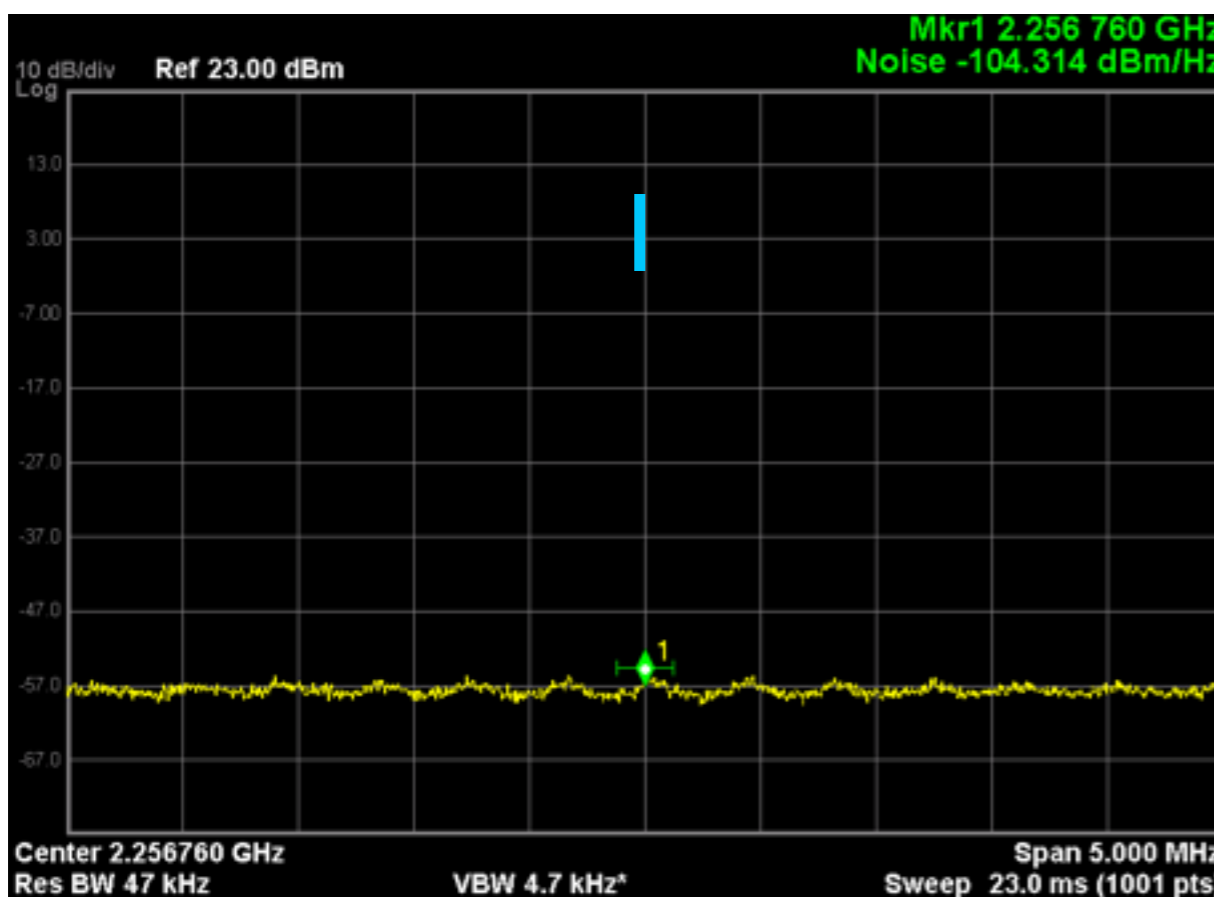


SNR

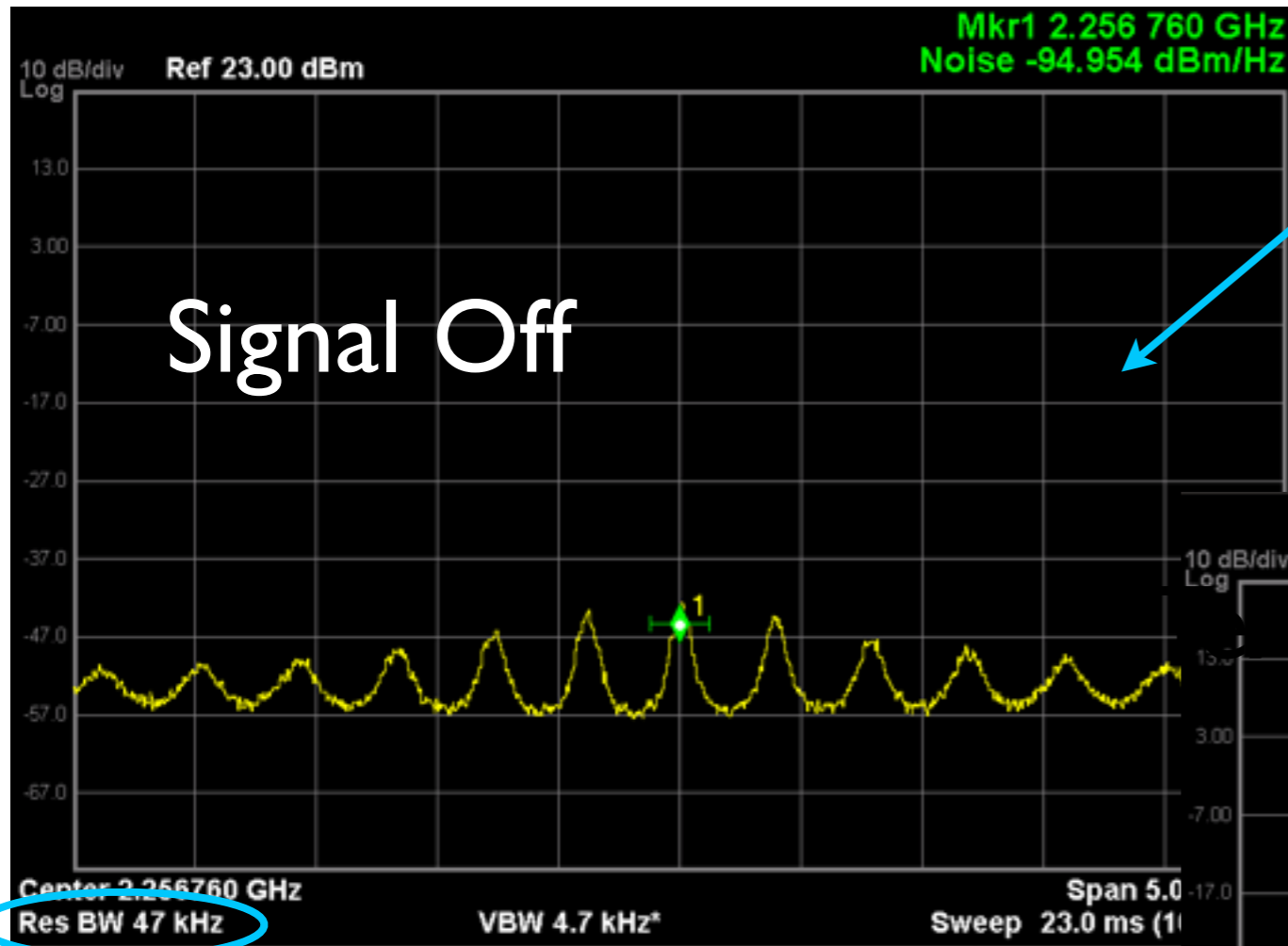


Q

Q

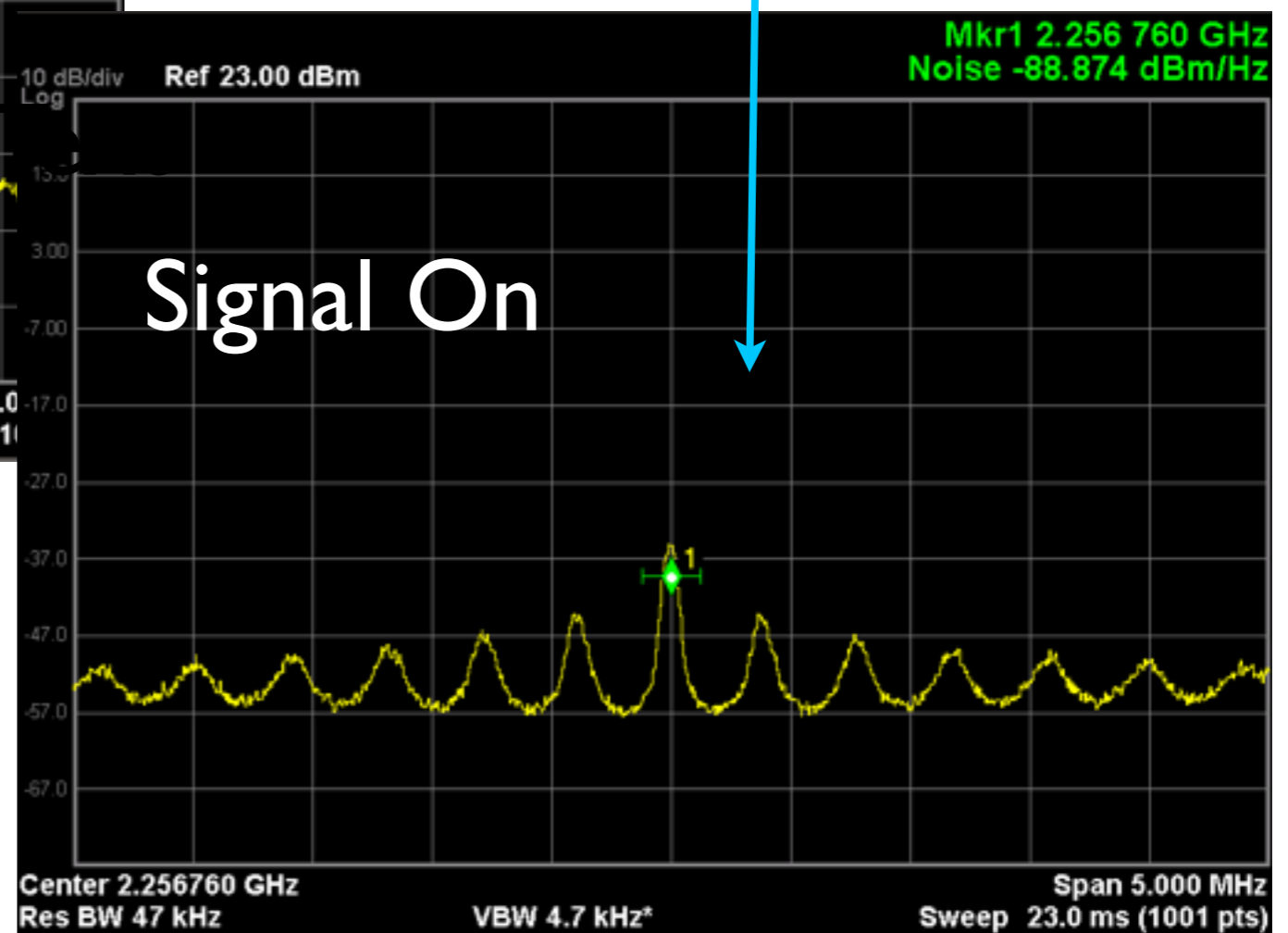


# spectrum analyzer trace



wiggles due to constructive/destructive interference at different frequencies

$$2\omega_0 t_0 / \pi \approx Q$$



the Q of these wiggles increases with feedback gain - need to make sure we are not filtering out our noise in each bin due to these peaks.